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JUPITER'S ATMOSPHERE*

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ABSTRACT. In this paper the two models of Jupiter's atmosphere, *viz.*, adiabatic and isothermal, have been considered. The variability of the period of rotation of the atmosphere depending on the latitude and the variation of gravity have both been taken into account. The datum level of Jupiter is the effective radiating and absorbing layer—probably a cloud layer—at a certain height in the atmosphere. It has the observed temperature of 150° Absolute. The authors have investigated the relation between pressure and density at any depth below the datum level and at any height above the datum level, in each of the two cases, *viz.*, (1) when the atmosphere consists of methane only, and (2) when it consists of a mixture of one part of methane and six parts of hydrogen. Even taking the atmosphere to be in adiabatic condition below the datum level and in isothermal condition above the datum level the authors have found that the total thickness of the atmospheric layer cannot, in any case exceed 1900 kms., and possibly it is below 1300 kms.

Before 1923, it was widely believed that the four great planets were very hot and that a large fraction of their volume was occupied by gas. In 1923 and 1924 H. Jeffries[1] investigated the physical constitution of the outer planets and suggested that they were cold. This conclusion was subsequently corroborated by observation which indicated a surface temperature of 150° absolute, for Jupiter. In a subsequent paper he further suggested that the Jupiter was formed of a rocky core covered by a thick layer of ice and that there was an atmosphere of more than 6000 kilometres in depth surrounding this icy layer. In 1934, Wildt[2] showed that an extensive atmosphere surrounding the surface of Jupiter will involve great density which makes it highly improbable that such an atmosphere exists there. Wildt gave 600 kilometres as the maximum depth that may possibly be attained by isothermal hydrogen atmosphere obeying gas laws.

In 1937, M. Peek[3] investigated the physical state of Jupiter's atmosphere. He found even smaller values for the maximum depth for the atmosphere than was indicated by Wildt.

The datum level of Jupiter is not the planet's solid surface, but it is the

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effective radiating and absorbing layer—probably a cloud layer—at a certain height in the atmosphere. The datum level does not rotate as a rigid body ; on the other hand it is observed that the period of rotation depends upon the latitude. Peek has taken the observed temperature of 150° Absolute to be the temperature of the datum level. He has taken three models for Jupiter's atmosphere, *viz.*, (1) the adiabatic, (2) the isothermal, and (3) a compromise in which an empirical relation is assumed between the depth and the lapse rate of temperature. In the first two cases for the purpose of numerical evaluation he has treated the atmosphere as though it was entirely composed of methane. In the third case not only he took methane in the unmixed state, but also in two mixed states with two different proportions of hydrogen. He investigated the relations between pressure, depth and density only at points below the datum level and not upwards in the atmosphere external to the datum level. He found that at a depth 25 kilometres below the datum level, atmosphere would probably become unrecognisable as such by the meteorologists.

In his investigation Peek has not considered the variability of the period of rotation of Jupiter's atmosphere. From observations of the spots near the pole the period is found to be about $9^h\ 55^m$ while for the equatorial spots it is about $9^h\ 50^m$. It appears that there is a sort of relative motion or general circulation in Jupiter's atmosphere. No satisfactory reason can be given for this relative motion as the theory of general circulation of the atmosphere is still very imperfect.

We have assumed the following formula for angular velocity at any point in the planet's atmosphere :

$$\omega = \omega_0 \left[1 + a_2 \left(\frac{r}{R}\right)^2 \sin^2\theta + a_4 \left(\frac{r}{R}\right)^4 \sin^4\theta \right], \qquad \ldots \quad (1)$$

where r is the distance of the point from centre of Jupiter and θ is the colatitude of the place. Here ω_0 is the angular velocity at points where $\theta = 0$ and R is the radius of the equatorial section of the datum level which is the stratum of the atmosphere that gives rise to the observed temperature 150° Absolute, and a_2 and a_4 are numerical constants. It may be mentioned here that spectroscopic observations[1] show that there is an equivalent atmosphere of one mile of methane and ten metres of ammonia at normal temperature and pressure above the visible photographic surface which we may take up as the datum level. The pressure at this level is calculated to be 0·335 atmosphere (earth's).

Some explanation seems to be necessary for the assumption of the above formula for the angular velocity. If the axis of z is the axis of rotation, we have the following equation for relative equilibrium :

$$\int \frac{dP}{\rho} = V + \int \omega^2 (x\,dx + y\,dy) = V + \tfrac{1}{2} \int \omega^2 d(x^2 + y^2), \qquad (2)$$

where V is the potential of the extraneous forces. In order that this equation may be integrable we must have a functional relation between P and ρ, and between ω and $\sqrt{x^2+y^2}$, *i.e.*, $r \sin \theta$.

We have to choose positive powers of $r \sin \theta$ in order that ω may not become infinite when $\theta = 0$. It will be seen later on that a_2 is negative and numerically greater than a_4 consequently within the atmosphere whose depth is small compared to the planet's radius, ω decreases as r increases along a radial line.

We have taken account of variation of gravity which was neglected by Peck. We have considered two models, (1) adiabatic, (2) isothermal. We have investigated the relation between pressure and density at any depth below the datum level and at any height above the datum level, in each of the two cases, *viz.*, (1) when the atmosphere consists of methane only, and (2) when it consists of a mixture of hydrogen and methane. We have calculated the possible maximum depth of the atmosphere below the datum level and also its possible maximum height above the datum level. The density at the outer boundary of the atmosphere is taken to be the inter-stellar density,[5] *viz.*, 10^{-26} c.g.s. units, and the density at the inner boundary of the atmosphere is taken to be the density of solid ammonia, *viz.*, 0·82 c.g.s. units, to allow for maximum possible depths. Alternative calculations have also been made by taking the density at the inner boundary to be that of solid methane, *i.e.*, 0·42 c.g.s. units in the first case and to be that of solid state of mixture, *i.e.*, 0·27 c.g.s. units in the second case.

For the period of rotation in different regions we have taken the following data for the year 1928, from two papers [6] published by A. Stanley Williams in 1934, the data for later years being not available :—

Region (south Latitude).	Mean Latitude (south).	Rotational Period.
Z°S to 16°S	8°S	$9^h\ 50^m\ 19{\cdot}2^s$
27°S to 37°S	32°S	$9^h\ 55^m\ 22{\cdot}6^s$
37°S to 55°S	46°S	$9^h\ 55^m\ 9{\cdot}8^s$

From the three rotational periods we calculate three values for ω which are assumed to hold at the mean latitudes of the three regions. These values are now substituted in formula (1). We thus get three equations which determine the values of a_2, a_4 and ω_0 :

$$a_2 = -\ {\cdot}06776 \text{ (numerical factor)}$$

$$a_4 = +\ {\cdot}05819 \text{ (,, ,,)} \qquad (3)$$

$$\omega_0 = \frac{2\pi}{35040} \text{ radians per sec.}$$

Our fundamental equations [7] of relative equilibrium are :—

$$\left.\begin{aligned} \frac{\partial P}{\partial r} &= \rho . \frac{\partial V}{\partial r} + \rho\omega^2 s\,(1-\mu^2) \\ \frac{\partial P}{\partial \mu} &= \rho . \frac{\partial V}{\partial \mu} - \rho\omega^2 r^2 \mu \end{aligned}\right\} \quad \ldots \quad (4)$$

Here r is the radial distance and $\mu = \cos\theta$, where θ is the polar angle, and V is the gravitational potential due to Jupiter's mass, and ω the angular velocity at any point of the atmosphere.

Taking Jupiter to be an oblate spheroid we find the expression for the gravitational potential [8] to be

$$V = \frac{GM}{r}\left[1 - \frac{3}{3} . \frac{P_2}{5} . \left(\frac{Re}{r}\right)^2 + \frac{3}{5} . \frac{P_4}{7} . \left(\frac{Re}{r}\right)^4 + \ldots\ldots\right]$$

where M is the mass of Jupiter, G the universal gravitation constant, R the semi-equatorial diameter, e the eccentricity of the Jupiter. Taking the semi-equatorial and semi-polar diameters of Jupiter [9] to be 71370 and 66620 kilometres respectively we find e^2 to be ·1287 and thus neglecting terms inside the bracket containing e^6 (which is of the order ·0001) and higher powers of e, we get

$$V = \frac{GM}{r}\left[1 - \frac{3}{3} . \frac{P_2}{5} . \left(\frac{Re}{r}\right)^2 + \frac{3}{5} . \frac{P_4}{7} . \left(\left(\frac{Re}{r}\right)^4\right)\right] \quad \ldots \quad (5)$$

We may remark here that as the extraneous forces are derived from a potential function, the equi-density surfaces are also equi-pressure and equi-temperature surfaces.

I. ISOTHERMAL CASE

1. We shall now consider the isothermal model. In this case

$$P = \frac{\beta T}{\mu}\rho = K\rho \quad \text{where } K = \left(\frac{\beta T}{\mu}\right)$$

where β is the universal gas constant and its value is $8{\cdot}26 \times 10^7$ c.g.s. units. T is taken to be 150° Absolute which is the observed temperature for the datum level.

Putting $P = K\rho$ and $\omega = \omega_0\left[1 + a_2\left(\frac{r}{R}\right)^2(1-\mu^2) + a_4\left(\frac{r}{R}\right)^4(1-\mu^2)^2\right]$

in our equations of relative equilibrium (4), we get

$$\frac{k}{\rho}\,\frac{\partial \rho}{\partial r} = \frac{\partial V}{\partial r} + \omega_0^2 r(1-\mu^2)\left[1 + a_2\left(\frac{r}{R}\right)^2(1-\mu^2) + a_4\left(\frac{r}{R}\right)^4(1-\mu^2)^2\right]^2$$

$$\frac{k}{\rho}\frac{\partial \rho}{\partial \mu}=\frac{\partial V}{\partial \mu}-\omega_0^2 r^2 \mu\left[1+a_2\cdot\left(\frac{r}{R}\right)^2(1-\mu^2)+a_4\left(\frac{r}{R}\right)^4(1-\mu^2)^2\right]^2$$

Multiplying the first of these equations by δr and the second by $\delta \mu$ and then adding and integrating, we get

$$k \log \rho = V+\omega_0^2 r^2\left[\frac{1-\mu^2}{2}+\frac{a_2^2}{2}(1-\mu^2)^2\left(\frac{r}{R}\right)^2+\frac{a_2^2}{6}.(1-\mu^2)^3\left(\frac{r}{R}\right)^4+\right.$$

$$\left.\frac{a_4}{3}(1-\mu^2)^3\left(\frac{r}{R}\right)^4+\frac{a_2a_4}{4}(1-\mu^2)^4\left(\frac{r}{R}\right)^6+\frac{a_4^2}{10}(1-\mu^2)^5\left(\frac{r}{R}\right)^8\right]+\text{constant} \quad \dots \quad (7)$$

is the complete solution of the equations of relative equilibrium. The arbitrary constant can be eliminated by substituting the given conditions. In each case we shall only consider the variation of density along a definite radial line. Hence in evaluating the arbitrary constant we shall not change the value of θ.

If $\rho=\rho_0$ when $r=r_0$, we get from (5) and (7)

$$k \log\left(\frac{\rho}{\rho_0}\right)=GM\left[\left\{\frac{1}{r}-\frac{1}{r_0}\right\}-\frac{P_2}{5}.(R^2c^2)\left\{\frac{1}{r^3}-\frac{1}{r_0^3}\right\}+\frac{3}{5}\,\frac{P_4}{7}.(Rc)^4\times\right.$$

$$\left.\left\{\frac{1}{r^5}-\frac{1}{r_0^5}\right\}\right]+\omega_0^2\left[\frac{(1-\mu^2)}{2}\left\{r^2-r_0^2\right\}+\frac{a_2}{2}\cdot\frac{(1-\mu^2)^2}{R^2}\left\{r^4-r_0^4\right\}\right.$$

$$+\frac{a_2^2}{6}\cdot\frac{(1-\mu^2)^3}{R^4}\left\{r^6-r_0^6\right\}+\frac{a_4}{3}\cdot\frac{(1-\mu^2)^3}{R^4}\left\{r^6-r_0^6\right\}$$

$$\left.+\frac{a_2a_4}{4}\cdot\frac{(1-\mu^2)^4}{R^6}\left\{r^8-r_0^8\right\}+\frac{a_4^2}{10}\cdot\frac{(1-\mu^2)^5}{R^8}\left\{r^{10}-r_0^{10}\right\}\right] \quad \dots \quad (8)$$

We shall take ρ_0 to be the density at the datum level which is assumed to be an equi-density surface for which the observed temperature is 150° Absolute and pressure is 0·335 atmosphere (Earth's).[10]

(A) We shall first take the atmosphere to consist of methane only—in this case $\mu=16$ being the molecular weight for methane.

1. *Equatorial Plane.*—Let us first calculate here the density in the equatorial plane. In the equatorial plane $\theta=\frac{\pi}{2}$, and we shall take $r_0=R$ (equatorial radius of Jupiter's surface) for datum level in this plane, for the sake of convenience in our calculations, since the error involved is of the order we are neglecting. We assume that the total height of Jupiter's atmosphere is small compared to its radius. For a point in the equatorial plane within the atmosphere at a radial distance ∂R from the datum level we take $r=R+\partial R$.

Neglecting $\left(\frac{\delta R}{R}\right)^2$ and higher powers of $\left(\frac{\delta R}{R}\right)$ we get after simplification, for the value of ρ at $r = R + \delta R$ in the equatorial plane from (5) and (7)

$$\log\left(\frac{\rho}{\rho_0}\right) = -\frac{GM}{KR}\left(\frac{\delta R}{R}\right)\left[1 + \frac{3e^2}{10} + \frac{9e^4}{56}\right] + \frac{\omega_0{}^2R^2}{K}, \left(\frac{\delta R}{R}\right)\left[1 + a_2 + a_4\right]^2 \dots \quad (9)$$

We take P_0 to be the pressure at the datum level and its value is 0·335 atmosphere (Earth's), as given by Peek. Now from the relation $P_0 = K\rho_0$, where $K = 7{\cdot}75 \times 10^8$ c.g.s. units (in this case $\mu = 16$) and 1 atmosphere [11] is equal to 1013600 dynes, and we find $\rho_0 = {\cdot}000438$ c.g.s. units.

We take the mass of Jupiter to be $\frac{1}{1047{\cdot}4}$ of Sun's mass as given by Russell.[12] Now Sun's mass is $1{\cdot}985 \times 10^{33}$ grammes as given by Eddington.[13] Thus Jupiter's mass is found to be $1{\cdot}896 \times 10^{30}$ grammes.

Here G is the universal gravitation constant and is equal to $8{\cdot}66 \times 10^{-8}$ c.g.s. units. Now for calculating the height of the outer boundary of the atmosphere we put $\rho = 10^{-26}$ c.g.s. units, which is known to be the inter-stellar density. Then our equation becomes, on substituting the numerical values,

$$-22{\cdot}6415 = \frac{\delta R \times (-3{\cdot}0403)}{10^6} \quad \text{(c.g.s. units)}$$

which gives the value of $\delta R = 74{\cdot}47 \times 10^5$ cms. $= 74{\cdot}47$ kilometres.

Now for finding the depth of the atmosphere below the datum level, we take ρ to be the density of solid ammonia which is 0·82 (c.g.s units) in order to allow for the maximum possible depth. Then substituting the numerical values as before we get

$$3{\cdot}2723 = -\frac{3{\cdot}0403}{10^6} \times \delta R$$

which gives $$\delta R = -10{\cdot}76 \times 10^5 \text{ cms.}$$

The negative sign stands for the depth which comes out to be 10·76 kilometres.

Next taking ρ to be the density of solid methane, *i.e.*, 0·42 c.g.s units, we get 9·81 kilometres as the depth.

(*ii*) *Colatitude* $\theta = 30°$.

We shall now calculate the density at any point in the radial line for which $\theta = 30°$. As before we shall calculate the radial height of the atmosphere above the datum level, for which $\rho = \rho_0$, and also the depth of the atmosphere below this point along the radius. Here also for the sake of convenience in our calculations we put $r_0 = r_1$, for the datum level, where r_1 is the radial distance from the centre of Jupiter of the point on its surface for which $\theta = 30°$, the error being of the order we are neglecting. For a point in the radial line $\theta = 30°$ within the atmosphere at a distance δr_1 from the datum level we take $r = r_1 + \delta r_1$.

Putting $r = r_1 + \delta r_1$ and $\theta = 30°$ in formula (7) and neglecting $\left(\frac{\delta r_1}{r_1}\right)^2$ and higher powers of $\left(\frac{\delta r_1}{r_1}\right)$, we get

$$\log\left(\frac{\rho_0}{\rho}\right) = \frac{GM}{Kr_1} \times \left(\frac{-\delta r_1}{r_1}\right)\left[1 - \frac{P_2}{5}\cdot\left(\frac{R}{r_1}\right)^2 \cdot 3e^2 + \frac{3}{5}\cdot\frac{P_4}{7}\cdot\left(\frac{R}{r_1}\right)^4 \cdot 5e^4\right]$$

$$+\frac{\omega_0^2 r_1^2(1-\mu^2)}{K} \times \left(\frac{\delta r_1}{r_1}\right)\left[1 + a_2\cdot\left(\frac{r_1}{R}\right)^2 (1-\mu^2) + a_4\cdot\left(\frac{r_1}{R}\right)^4 (1-\mu^2)^2\right]^2 \quad \ldots \quad (10)$$

The values of r, P_2 and P_4 are found to be

$$\left.\begin{aligned} r_1 &= 6{\cdot}772 \times 10^9 \text{ cms.} \\ P_2 &= {\cdot}025 \\ P_4 &= {\cdot}0225 \end{aligned}\right\} \quad \ldots \quad (11)$$

Substituting the numerical values from (3) and (11) and putting $\rho = 10^{-26}$ (c.g.s units) for the outer boundary of the atmosphere, we have

$$-22{\cdot}6415 = \frac{-3{\cdot}2678 \times \delta r_1}{10^6} \text{ (c.g.s. units)}$$

$$\therefore \quad \delta r_1 = \frac{22{\cdot}6415 \times 10^6}{3{\cdot}2678} = 6{\cdot}928 \times 10^6 \text{ cms.}$$

$$= 69{\cdot}28 \text{ kms.}$$

i.e., the height comes out to be 69·28 kms. Similarly putting $\rho = 0{\cdot}82$ for the inner boundary we get, for the depth of the atmosphere,

$$3{\cdot}2723 = \frac{-3{\cdot}2678 \times \delta r_1}{10^6} \text{ (c.g.s. units)}$$

$$\therefore \quad \delta r_1 = \frac{10^6 \times 3{\cdot}2723}{-3{\cdot}2678} = -1{\cdot}001 \times 10^6 \text{ cms.}$$

$$= -10{\cdot}01 \text{ kms.}$$

Therefore the depth is 10·01 kilometres.

Again taking $\rho = 0{\cdot}42$, density of solid methane we get the equation

$$2{\cdot}9817 = \frac{-3{\cdot}2678 \times \delta r_1}{10^6}$$

or
$$\delta r_1 = -9{\cdot}12 \times 10^5 \text{ cms.} = -9{\cdot}12 \text{ kms.}$$

i.e., the depth is 9·12 kms.

(B) Next let us suppose the atmosphere to consist of a mixture of 1 part of methane and 6 parts of hydrogen.

In this case the value of μ becomes 4 instead of 16. We substitute the new value of μ in our equations and proceed as before.

(*i*) *Equatorial Plane*.—In the equatorial plane, the height of the atmosphere above the datum level is found to be 297·88 kms. The depth below the datum level is found to be 43·04 kilometres, if the density of inner boundary is taken to be 0·82, *i.e.*, the density of solid ammonia.

Next if we take the density of the inner boundary to be the density of solid state of mixture, *i.e.*, 0·27, we find the depth to be 36·71 kilometres.

(*ii*) *Colatitude* $\theta = 30°$.—Considering the extent of atmosphere in the radial line $\theta = 30°$, and proceeding as before, we find that the height of the atmosphere above the datum level is 277·12 kilometres and the depth below the datum level is 40·04 kms., if we take the density of the inner boundary to be the density of solid ammonia.

Again if we take the density of the inner boundary to be the density of solid state of the mixture, *i.e.*, 0·27, then we find the depth to be 34·15 kilometres.

II. ADIABATIC MODEL

(C) In considering the adiabatic model also, we shall first assume that the atmosphere consists of methane alone. The relation between pressure and density in this case is

$$P = K_2 \rho^\gamma \qquad \text{where} \quad K_2 = \left(\frac{\beta T}{\mu}\right)^\gamma P^{1-\gamma} \;;$$

here γ denotes the ratio of specific heats. We shall put $\gamma = 1 + \frac{1}{\lambda}$. For methane $\mu = 16$ and now taking as before the value for the pressure P_0 at the datum level in the equatorial plane, to be ·335 atmosphere (Earth's) and $T = 150°$ Absolute, and substituting numerical values in (12), we find that $K_2 = 7{\cdot}884 \times 10^9$ c.g.s. units.

Now putting $P = K_2 \rho^{1+\frac{1}{\lambda}}$ in the fundamental equations (4) and integrating as before, we get

$$K_2(\lambda+1)\rho^{\frac{1}{\lambda}} = \frac{GM}{r}\left[1 - \frac{3}{3} \cdot \frac{P_2}{5} \cdot \left(\frac{Re}{r}\right)^2 + \frac{3}{5} \cdot \frac{P_4}{7} \cdot \left(\frac{Re}{r}\right)^4\right] + \omega_0^2 r^2$$

$$\left[\frac{(1-\mu^2)}{2} + \frac{a_2}{2}\cdot\left(\frac{r}{R}\right)^2(1-\mu^2)^2 + \frac{a_1}{3}\cdot\left(\frac{r}{R}\right)^4(1-\mu^2)^3 + \frac{a_2^2}{6}\cdot\left(\frac{r}{R}\right)^4(1-\mu^2)^3\right.$$

$$\left. + \frac{a_2 a_1}{4}\cdot\left(\frac{r}{R}\right)^6(1-\mu^2)^4 + \frac{a_1^2}{10}\cdot\left(\frac{r}{R}\right)^8(1-\mu^2)^5\right] + \text{constant} \qquad \ldots \quad (13)$$

As before the constant can be evaluated for a definite radial line from the given conditions. In this case the right hand side of equation (8) remains the same and for the left-hand side we shall have $k_2\ (1+\lambda)\left(\rho^{\frac{1}{\lambda}} - \rho_0^{\frac{1}{\lambda}}\right)$ instead of $k \log\left(\frac{\rho}{\rho_0}\right)$.

(*i*) *Equatorial Plane.*—We shall first find the extent of the atmosphere in the equatorial plane. Putting

$$\rho = \rho_0, \quad r_0 = R, \quad \theta = \frac{\pi}{2} \quad \text{and} \quad r = R + \delta R$$

and proceeding exactly as in the isothermal case, we get

$$K_2\left(1+\lambda\right)\left(\rho^{\frac{1}{\lambda}} - \rho_0^{\frac{1}{\lambda}}\right) = -23{\cdot}56 \times 10^2 \times \delta R.$$

On substituting the values of K_2 and λ, we get

$$\left(\rho^{\frac{1}{\lambda}} - \rho_0^{\frac{1}{\lambda}}\right) = \frac{-\delta R}{10^6} \times {\cdot}06896.$$

For finding the height of the outer boundary, we put $\rho = 10^{-26}$ (c.g.s. units), so that $\left(\rho^{\frac{1}{\lambda}} - \rho_0^{\frac{1}{\lambda}}\right) = {\cdot}09828$ (c.g.s. units), and the height is given by $\delta R = \frac{{\cdot}09828 \times 10^6}{{\cdot}06896} = 14{\cdot}25 \times 10^5$ cms $= 14{\cdot}25$ kms. Again for finding the depth of the atmosphere below the datum level, we first put $\rho=0{\cdot}82$, which gives

$\left(\rho^{\frac{1}{\lambda}}-\rho_0^{\frac{1}{\lambda}}\right)={\cdot}8438$ so that the depth $=-\delta R=\dfrac{{\cdot}8438\times10^6}{{\cdot}06896}=122{\cdot}4\times10^5$ cms. $=122{\cdot}4$ kms.

Next taking ρ to be density of solid methane $0{\cdot}42$ we find that

$\left(\rho^{\frac{1}{\lambda}}-\rho_0^{\frac{1}{\lambda}}\right)={\cdot}6726$ (c.g.s. units) so that the depth $(-\delta R)$ comes out to be $97{\cdot}54$ kms.

(*ii*) *Colatitude* $\theta=30^\circ$.—Next let us find the extent of atmesphere in colatitude $\theta=30^\circ$. Proceeding exactly as in the isothermal case and substituting

the values of K_2 and λ we get $\left(\rho^{\frac{1}{\lambda}}-\rho_0^{\frac{1}{\lambda}}\right)=-\dfrac{\delta r_1\times{\cdot}07413}{10^6}$ where δr_1 has the same meaning as before.

Putting $\rho=10^{-26}$ c.g.s. units, for the height of the outer boundary, we get

$$\delta r_1=\frac{10^6\times{\cdot}09828}{{\cdot}07413}=13{\cdot}26\times10^5\text{ cms.}=13{\cdot}26\text{ kms.}$$

Similarly putting $\rho=0{\cdot}82$ for the density of the inner boundary, we get, for the depth below the datum level.

$$-\delta r_1=\frac{10^6\times{\cdot}8438}{{\cdot}07413}=113{\cdot}9\times10^5\text{cms.}=113{\cdot}9\text{ kms.}$$

Again putting $\rho=0{\cdot}42$ for the density of the inner boundary we find the depth below the datum level to be $90{\cdot}74$ kms.

(D) Next let us consider the case of the atmosphere consisting of a mixture of 1 part of methane and 6 parts of hydrogen. In this case the value of

$\mu=4$ and $1+\dfrac{1}{\lambda}=1{\cdot}4$. Consequently $\lambda=\dfrac{5}{2}$.

With these new values for μ and λ we find that value of K_2 in this case is $1{\cdot}191\times10^{11}$ (c.g.s. units).

(*i*) *Equatorial Plane.*—Our equation giving the density in the equatorial plane remains the same in form as in C (*i*) except that the values of K_2 and λ will be changed, and we get

$$K_2\,(1+\lambda)\left(\rho^{\frac{1}{\lambda}}-\rho_0^{\frac{1}{\lambda}}\right)=-23{\cdot}56\times10^2\times\delta R$$

Substituting the numerical values of K_2 and λ and putting $\rho=10^{-26}$ c.g.s. units, we get, for the height of the atmosphere above the datum level,

$$\delta R = \frac{\cdot 04535 \times 10^8}{\cdot 5657} = 8\cdot 019 \times 10^6 \text{ cms.} = 80\cdot 19 \text{ kms.}$$

Similarly for the depth below the datum level. we get

$$-\delta R = \frac{\cdot 8784 \times 10^8}{\cdot 5657} = 15\cdot 53 \times 10^7 \text{ cms.} = 15\cdot 53 \text{ kms.}$$

by taking $\rho = 0\cdot 82$ (c.g.s. units) at the inner boundary.

Next putting the density at the inner boundary to be $0\cdot 27$ (c.g.s. units), *i.e.*, the density of solid state of the mixture, we get $-\delta R = \frac{\cdot 5471 \times 10^8}{\cdot 5657} = 967\cdot 4$ kms. as the depth below the datum level.

(*ii*) *Colatitude* $\theta = 30°$.—Similarly we can find the density at any point in the radial line $\theta = 30°$, by proceeding exactly in the same way as in the isothermal case. We get

$$K_2(1+\lambda)\left(\rho^{\frac{1}{\lambda}} - \rho_0^{\frac{1}{\lambda}}\right) = -25\cdot 33 \times 10^2 \times \delta R\ ;$$

substituting for K_2 and λ, we get $\left(\rho^{\frac{1}{\lambda}} - \rho_0^{\frac{1}{\lambda}}\right) = \frac{-\cdot 608 \times \delta R}{10^8}$.

Putting $\rho = 10^{-26}$ (c.g.s. units), we get for the height of the atmosphere above the datum level, $\delta R = \frac{\cdot 04535 \times 10^8}{\cdot 608} = 74\cdot 59 \times 10^5$ cms. $= 74\cdot 59$ kms.

Similarly putting $\rho = 0\cdot 82$ (c.g.s. units) for the density of the inner boundary, we get $-\delta R = \frac{\cdot 8784 \times 10^8}{\cdot 608} = 14\cdot 45 \times 10^7$ cms. $= 1445$ kms. as the depth below the datum level.

Again putting $\rho = 0\cdot 27$ (c.g.s. units) for the density of the inner boundary we get $-\delta R = \frac{10^8 \times \cdot 5471}{\cdot 608} = 899\cdot 7$ kms. as the depth below the datum level.

We now give below our results in a tabular form.

TABLE I

Atmosphere of Methane alone

	Isothermal		Adiabatic	
	Equatorial plane.	Colatitude $\theta = 30°$.	Equatorial plane.	Colatitude $\theta = 30°$.
Height in kms. above the datum level	74·47	69·28	14·25	13·26
Depth in kms below the datum level, taking the density at the inner boundary to be that of solid methane.	9·81	9·12	97·54	90·74
Depth in kms below the datum level, taking the density at the inner boundary to be that of solid ammonia*	10·76	10·01	122·4	113·9

TABLE II

Atmosphere consisting of a mixture of 1 part of Methane and 6 parts of Hydrogen

	Isothermal		Adiabatic	
	Equatorial plane.	Colatitude $\theta = 30°$	Equatorial plane.	Colatitude $\theta = 30°$.
Height in kms. above the datum level	297·88	277·12	80·19	74·59
Depth in kms. below the datum level, taking the density at the inner boundary to be that of solid state of mixture.	36·71	34·15	967·4	899·7
Depth in kms below the datum level, taking the density at the inner boundary to be that of solid ammonia.*	43·04	40·04	1553	1445

Assuming the atmosphere to consist of a mixture of 1 part of methane and 6 parts of hydrogen, the probable thickness of the atmosphere in the equatorial plane appears to be about 334 kms. under isothermal condition and 1050 kms.

* To allow for maximum possible depths we have also calculated them by assuming the density at the inner boundary of the atmosphere to be that of solid ammonia, although only traces of ammonia are revealed by the spectroscope.

under adiabatic condition. The datum level in Jupiter's atmosphere probably consists of a cloud layer as mentioned before ; so from analogy of terrestrial conditions, it will not perhaps be unreasonable to assume that from the inner boundary (solid surface of Jupiter) of the atmosphere to the datum level, the atmosphere is adiabatic and from the datum level to the outer boundary of the atmosphere the atmosphere is isothermal. With this assumption the total thickness of Jupiter's atmosphere will be about 1200 kilometres. This is quite a plausible figure. We cannot extend our terrestrial analogy too far and we have no evidence to show that there is any region in Jupiter's atmosphere in which temperature steadily increases with height due to ionization.

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